# A Gallon of Pentagons

There are 15 known types of tiling pentagons. You now have 12 of each type, which totals roughly one gallon.

For a shape to *tile*, it needs to be able to cover a flat surface without gaps or overlaps—just like tiles on a bathroom floor. Twelve pentagons is enough for most people to convince themselves that they have figured out a pattern that would allow them to continue covering a flat surface of any size. If you find yourself needing more you can trace onto paper or cardboard, or you can order more wooden ones and we'll gladly make them for you.

Tiling pentagons have a long history, and only recently did we learn that there are exactly 15 types. The most recent tiling pentagon (the 15th) was discovered in 2015. The French mathematician Michaël Rao proved in 2017 that there are no more left to find.

Even when you know the pentagons will tile, figuring out how can be quite challenging. This sheet describes the defining characteristics of each type of pentagon, which is useful information as you try to tile them. Some types (such as 1) allow for a wide variety of seemingly quite different pentagons. Others (such as 15) can only vary by size—you could tile with bigger ones, but they would look exactly like the smaller ones.

The types are not exclusive. It is possible for a pentagon to be of more than one type. Such a pentagon can tile in two different ways.

Some pentagons are much more difficult to solve than others.

The numbers 1—15 correspond roughly to their order of discovery, but not perfectly. They also do not correspond to order of difficulty, although many of the smaller-numbered ones are easier for most people to solve than the larger–numbered ones. Pentagon number 15 is very challenging.

If you would like to know more about tiling pentagons, here are some resources for further reading.

"**Tiling with Convex Polygons**" is a chapter in the book *Time Travel and Other Mathematical Bewilderments* by Martin Gardner. Gardner wrote a recreational math column for many years in *Scientific American.* This book is out of print, but easily obtainable used online, and is in many large libraries.

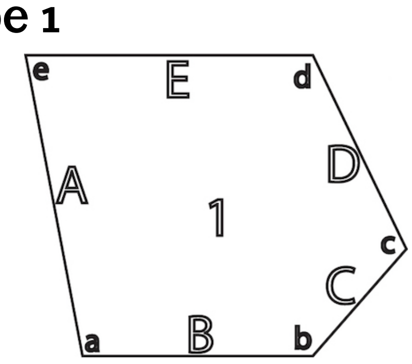
**Marjorie Rice** was not a trained mathematician, but on reading Gardner's column, she found several new tiling types. Her website has some beautiful examples of her work.
*https://sites.google.com/site/intriguingtessellations/home*

**Doris Schattshneider** is a professional mathematician, retired from Moravian College. She collaborated with Marjorie Rice, and has written and published many things related to tilings, including tiling pentagons. Google her.

"**A Toast! To Type 15!**" is an article celebrating the new pentagon shortly after its announcement. The article is in the November 2015 issue of *Math Horizons*. This journal is published by the Mathematical Association of America and aimed at an undergraduate college math audience. You should be able to find a copy in any college library, or through a public library's interlibrary loan.

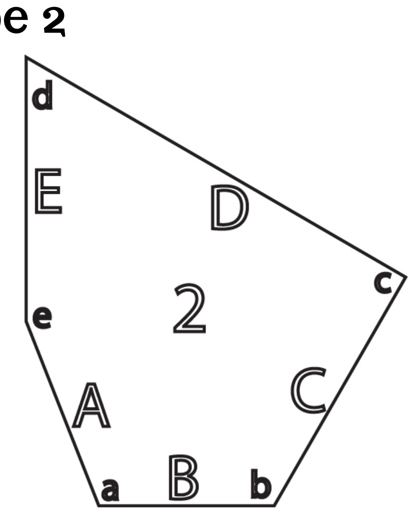
The Wikipedia article on "**Pentagonal Tiling**" gets very technical very quickly, but is the best, most easily available, comprehensive resource for background and information about the mathematics of tiling pentagons. It is also the place where any future developments will be tracked.

## Type 1



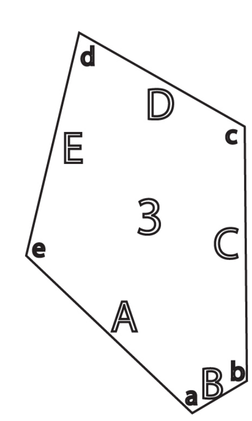

Any pentagon with two parallel sides is type 1.

## Type 2



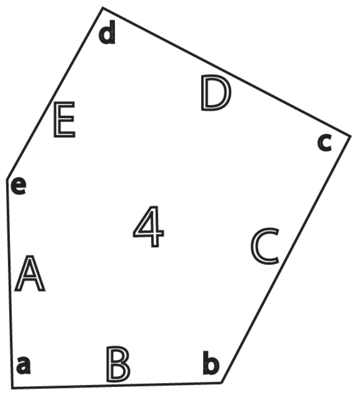

Angles **b** and **d** have measures that add up to 180°. Sides **C** and **E** are the same length as each other.

## Type 3



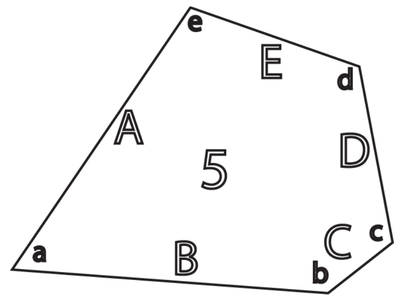

Angles **b**, **c**, and **e** each measure 120°. Sides **B** and **D** combine to be the same length as side **C**. Sides **A** and **E** are the same length as each other.

## Type 4



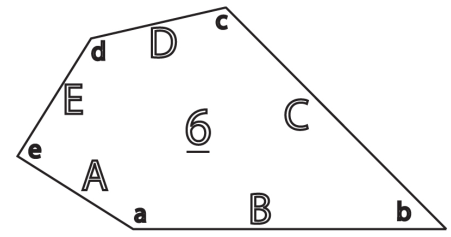

Angles **a** and **c** are right angles. Sides **A** and **B** are the same length as each other. Sides **C** and **D** are the same length as each other.

## Type 5



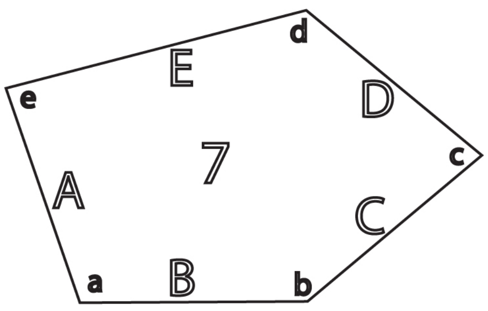

Angle a measures 60°. Angle d measures 120°. Sides **A** and **B** are the same length as each other. Sides **D** and **E** are the same length as each other.

## Type 6



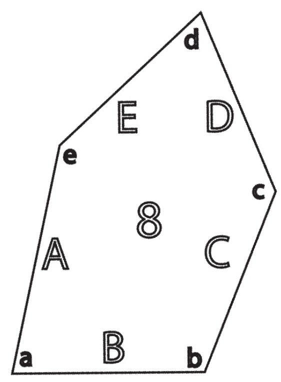

Angles **b** and **d** have measures that add up to 180°. Angle **e** is twice the size of angle **b**. Sides **A**, **D**, and **E** are the same length as each other. Sides **B** and **C** are the same length as each other.

## Type 7



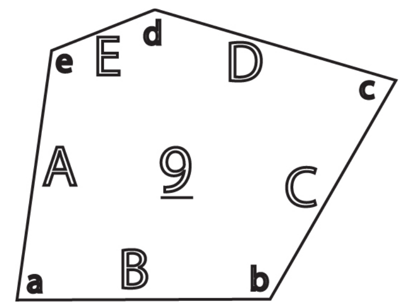

If you double the measure of angle **b**, and add it to **c**, you get 360°. If you double the measure of angle **d**, and add it to **a**, you get 360°. Four of the five sides are the same length: **A**, **B**, **C**, and **D**.

## Type 8



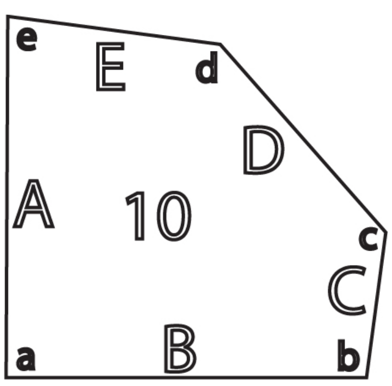

If you double the measure of angle **b**, and add **c**, you get 360°. If you double the measure of angle e, and add d, you get 360°. Four of the five sides are the same length: **B**, **C**, **D**, and **E**.

## Type 9



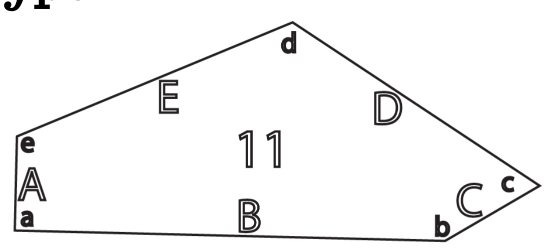

If you double the measure of angle **e**, and add **b**, you get 360°. If you double the measure of angle **d**, and add **c**, you get 360°. Four of the five sides are the same length: **A**, **B**, **C**, and **D**.

## Type 10



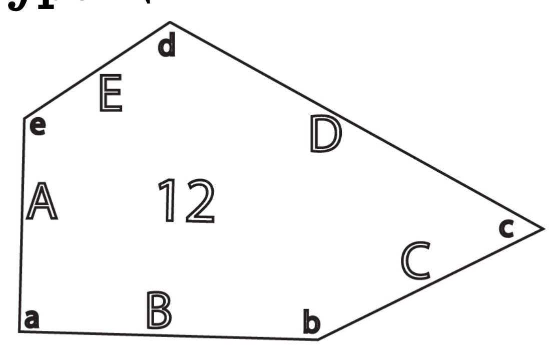

Angle **a** measures 90°. The measures of angles **b** and **e** sum to 180°. If you double the measure of angle **c**, and add **b**, you get 360°. Sides **A** and **B** are the same length as each other, and the lengths of sides **C** and **E** add up to this length.

## Type 11



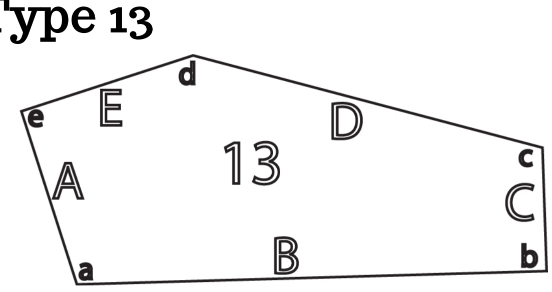

Angle **a** measures 90°. If you double the measure of angle **b**, and add **c**, you get 360°. The measures of angles **c** and **e** sum to 180°. Sides **D** and **E** are the same length as each other. If you double the length of side **A** and add the length of side **C**, you get the same length as **D** and **E**. `

## Type 12



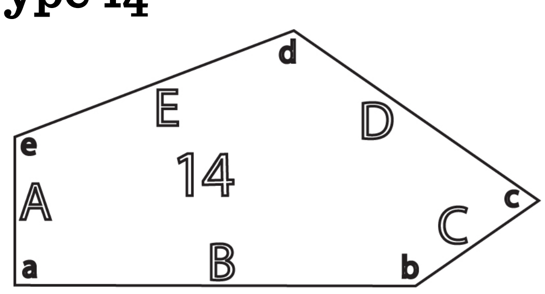

Angle **a** is 90°. If you double the measure of angle **b**, and add the measure of angle **c**, you get 360°. Angles **c** and **e** sum to 180°. Side **D** is twice as long as side **A**. Also, side **D** is the same length as the sum of sides **C** and **E**.

## Type 13



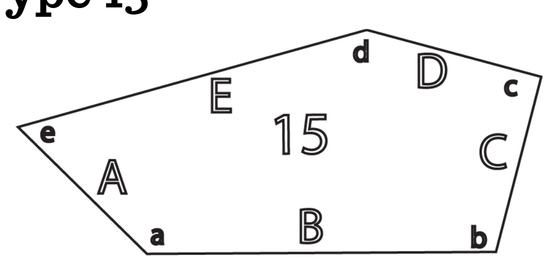

Angles **b** and **e** measure 90°. If you double the measure of angle **a**, and add **d**, you get 360°. Sides **A** and **E** are the same length as each other. Side **D** is twice as long as each of them.

## Type 14




All of the angle measures are specified. Angle **a** is 90°. Angle **b** is 145.54°. Angle **c** is 69.32°. Angle **d** is 124.66°. Angle **e** is 110.68°. Sides **D** and **E** are the same length as each other. Sides **A** and **B** are the same length as each other, and half the length of sides **D** and **E**.

## Type 15




All of the angle measures are specified exactly. Angle **a** is 135°. Angle **b** is 105°. Angle **c** is 90°. Angle **d** is 150°. Angle **e** is 60°. Sides **A**, **C**, and **D** are the same length as each other. Side **E** is twice as long as them.